Państwo i Społeczeństwo

2016 (XVI) nr 2

e-ISSN 2451-0858
ISSN 1643-8299

Mara Cotič, Darjo Felda

PRIMARY, TEACHERS' PROFESSIONAL QUALIFICATION FOR TEACHING MATHS

Kwalifikacje zawodowe do nauczania matematyki nauczycieli wczesnej edukacji

Autorzy artykułu poszukują odpowiedzi na pytanie, czy w dzisiejszych czasach system nauczania matematyki jest wystarczająco skuteczny, aby jego efektem był matematycznie wykształcony człowiek. Autorzy postulują, by do nauczania matematyki zostały włączone strategie rozwiązywania problemów powiązane z życiowym doświadczeniem. W artykule pojawia się też silna sugestia, że uczniowie podczas nauczania matematyki powinni przechodzić przez wszystkie etapy realistycznego rozwiązywania problemów, dzięki czemu rozwiną się ich umiejętności matematyczne i powiązanie matematyki z życiem codziennym, a jednocześnie taki sposób kształcenia matematycznego zapewni sens pojęć matematycznych i matematycznych procedur. Autorzy uważają ponadto, że nauczyciele umiejętności matematycznych powinni zmienić swój punkt ciężkości z rodzaju nauczania, w którym jedynie przekazują wiedzę matematyczną na sposób nauczania, w którym uczniowie będą mieli możliwość odkrywania i budowania swojego systemu wiedzy.
Słowa kluczowe: nauczanie matematyki, rozwiązywanie realnych problemów, umiejętności matematyczne

Introduction

According to contemporary studies the main characteristic of the concept of education, school, mathematics education and learning is developing thinking skills and diminishing the knowledge acquisition[1]. The focus lies on acquiring knowledge that can be more or less directly implemented to solve everyday problem situations. Also UNESCO and EU Ministers of Education have issued some recommendations that require increased student activity, development of creativity, independence and thinking skills and training pupils for problem solving. Teaching that encourages memorization, listening and repetition of what pupils have learned should be omitted. Teachers should encourage the development of problem solving skills and logical thinking skills.

The process of learning mathematics is an active process of knowledge acquisition, a process of interaction through which pupils build new cognitive structures and develop their cognitive and other abilities, a process in which the role of the teacher is to help pupils acquire new and restructure prior knowledge, and not a process in which pupils just passively adopt certain mathematical content and acquire ready-made knowledge[2]. Mathematics education should be based primarily on a realistic approach in which pupils learn in real context, build their knowledge on real-life situations and develop mathematical thinking[3]. The acquired knowledge must be gradually sufficiently solid and structured, otherwise it can't be a suitable basis for later learning, in view of the cumulative character of mathematical knowledge[4].

The teaching of mathematics should develop the following aspects of learning:

- exploring,
- problem solving,
- creative thinking,
- data processing,
- logical reasoning, and
- estimation of results.

Creative exploration and problem solving is an excellent way of develop-

---

[1] V.R. Jacobs, M.L. Franke, T.P. Carpenter, L. Levi, D. Battey, *Professional development focused on children's algebraic reasoning in elementary schools*, "Journal for Research in Mathematics Education" 2007, Vol. 38 (3), p. 258–288.

[2] S. Maričić, K. Špijunović, *Aktivnost učenika i razvijanje kritičkog mišljenja u početnoj nastavi matematike*, [in:] *Aktivnosti učencev v učnem procesu*, ed. D. Hozjan, Koper 2015, pp. 281–290.

[3] K. Špijunović, S. Maričić, *Development of pupils' mathematical thinking as a goal and task of the initial teaching of Mathematics. The modern society and education – The VI International Balkan Congress for Education and Science*, Skopje 2011, pp. 975−981; B. Lazić, S. Maričić, *Propaedeutic formation of the concept of fraction in elementary mathematics education*, [in:] *Developing mathematical language and reasoning* (Proceeding of International Simposium Elementary Math Teaching), eds. J. Novotna, H. Moraova, Prague 2015, pp. 212–221.

[4] UNESCO, *Challenges in basic mathematics education*, Paris 2012.

ing mathematical concepts, as pupils raise questions and presume possible conclusions, select strategies and representations, use their thinking skills, prove or reject assertions, critically review, verify, and assess their work, and develop patience and persistence to reach a solution.

Solving realistic problems

The most frequently realistic problems are solved with the mathematisation of a non-mathematic situation, i.e.:

- building a mathematical model according to the corresponding realistic situation or situation from everyday life;
- solving the built mathematical problem;
- transferring the solution of the mathematical problem that corresponds to the mathematical model into realistic environment.

Solving a realistic problem can be presented with a diagram, as evident in Figure 1[5].

To understand why many children struggle with text-based tasks used to present mathematical problems, though they require 'solely' the application of a learnt process, it is necessary to investigate how they develop a relevant mathematical model.

At the beginning of education, pupils do not experience mathematics as a tool for solving realistic problems, but rather as some kind of representation of a variety of different things or objects, e.g. balls, blocks, sticks, etc. with numbers. The modelling situation is actually reversed for them and indicates the process of visualisation or illustration as is seen on figure 2[6].

The focus is not on manipulating mathematical objects, but rather on manipulating real objects. It means that 'standard' text-based tasks are not really suitable for developing mathematical modelling skills, because the text itself indicates the choice of the right mathematical operation or process, making the search for the links between the real problem situation and mathematics unnecessary. Tougher problems that present a challenge for pupils are more suitable for developing mathematical literacy.

It is better for pupils in early education to solve problems in small groups. Not only do they find relevant strategies through constructive cooperation, but in their discussion they also pinpoint potential critical points for which they seek an optimal solution. In this way, any individual can gain experience and develop mathematical literacy.

[5] G. Müller, E. Wittmann, *Der Mathematikunterricht in der Primarstufe*, Braunschweig 1984.

[6] A. Peter-Koop, *Fermi problems in primary mathematics classrooms: Pupils' interactive modelling processes*, [in:] *Mathematics education for the third millennium: Towards 2010*, eds. I. Putt, R. Faragher, M. McLean, Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Sydney 2004, pp. 454–461.

Figure 1. Solving a realistic problem[7]

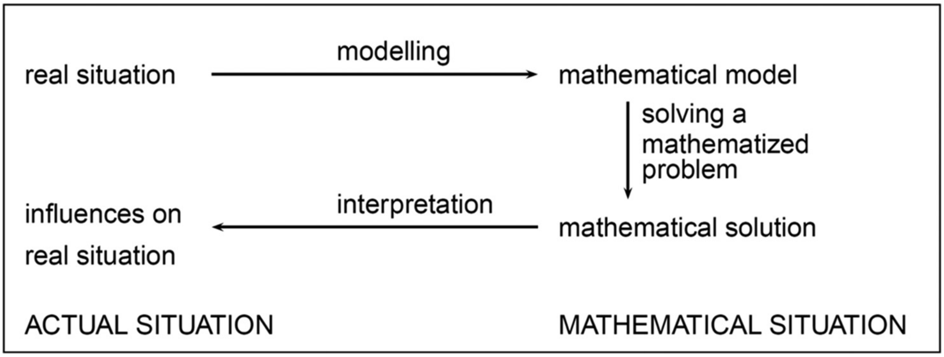
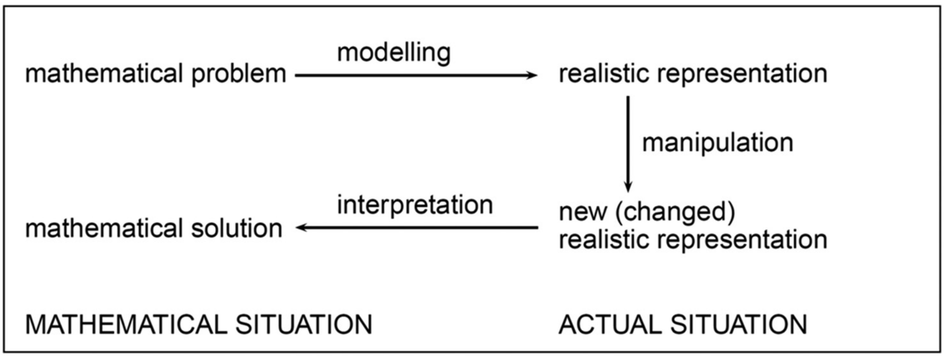


Figure 2. Solving a mathematical problem with the use of real representation[8]



Real-world problems adjusted to pupils' age should not contain too much numeric data so as not to discourage pupils from analysing the described problem situation and make them calculate or search for a calculation or other algorithms. If data is provided in a descriptive mode, pupils will be inclined to assess, estimate, seek and collect relevant data. Problems must be 'open' enough to make pupils take reasoned decisions in the process of solving based on the mathematical model they use[9].

For the development of mathematical literacy it is necessary to put emphasis on the real situation and not on the problems that ought to be pruned of "unnecessary" situational elements. Of course, pupils in their early education should be properly assisted or guided when handling realistic problems. They are supposed to create the solving process independently as much as possible, so they

[7] G. Müller, E. Wittmann, *op. cit.*

[8] A. Peter-Koop, *op. cit.*

[9] *Ibidem*.

should be given less complex problems from which they will manage to extract the information and data necessary to create an effective and simple situation model. In no case should pupils be prevented from solving realistic problems just because their knowledge is considered not effective enough. They must constantly develop and improve relevant strategies of solving realistic problems and include their knowledge and skills in them.

The teaching of mathematics and solving realistic problems

Solving realistic problems is important because of the perception of mathematical models in our everyday environment that thus

- helps the child and the adolescent in understanding the world;
- supports the learning of mathematics in the sense of motivation, formation of concepts, and understanding terms;
- helps in the development of mathematical competences and the corresponding skills;
- helps in the understanding of mathematics as a discipline[10].

This makes mathematics more meaningful for the pupil, as usual textual tasks are the most frequently just an artificial "real-life" disguise for mathematical problems and do not represent the learner any challenge—for the pupil engagement with these assignments is a boring mathematisation of mathematics. Emphasising pupils need to offered textual tasks with enough interesting and enough challenging content from their everyday has become one of the basic guidelines of teaching mathematics. If a realistic problem is given as a textual task, it is necessary we identify ourselves with the corresponding problem situation and use the knowledge and experience from everyday life.

This approach to teaching mathematics is also difficult for teachers, as in addition to mathematical knowledge it also requires the knowledge and experience the individual acquires in everyday life. Teachers usually carefully plan their teaching and foresee the exact course of the lesson nearly to the minute; they even put down the questions and the answers they expect from their pupils. Teaching with solving realistic problems cannot be equally predictable; the teaching becomes more "open". This is why many teachers are afraid the pupils would not acquire the knowledge or attain the corresponding learning goals. They believe they can determine whether the learning goal has been met only with "conventional" teaching methods; they find problem oriented teaching "dispersed" and time consuming; they have no control over it.

The aim of mathematics learning and teaching must be to raise the level of mathematical literacy. Pupils should not be exposed only to unrealistic, simpli-

---

[10] W. Blum, R. Borromeo Ferri, *Mathematical Modelling: Can It Be Taught And Learnt?*, "Journal of Mathematical Modelling and Application" 2009, Vol. 1 (1), pp. 45–58.

fied, 'abstract' and 'artificialized' problems, but rather to problems that derive from their real situations. Therefore, in the mathematics classroom pupils should encounter problems with:

- redundant data,
- insufficient data,
- multiple solutions, and
- contradicting data.

Let us look at a problem situation that can happen in daily life. The description includes redundant information independent of the information needed to solve the problem situation:

The shop sold T-shirts for 20 euros, trousers for 45 euros and sports shoes for 60 euros. Ajda bought a T-shirt and paid with a €50 banknote. How many euros of change did she receive?

The next problem has an implicitly written solution for Ajda, her classmates, the teacher and some other people. It is implicitly stated where the problem situation takes place, so information about the number of pupils in Ajda's class needs to be found. However, the problem is unsolvable for those who are unable to obtain this information or do not even know which class is in question:

Ajda brought sweets to school for her birthday. She gave one sweet to each pupil and the teacher, and there were 3 sweets left. How many sweets did she bring to school?

As for problems with multiple solutions, most typical ones are about determining distances between places, shopping (limited by a certain amount of money) or numbers of animals (some with two, others with four legs). Here are two examples:

Maja had 200 euros. In the supermarket T-shirts cost 20 euros, jumpers 45 euros, pullovers 90 euros and trousers 110 euros. What could Maja afford?

There were sheep, cows and chickens on the pasture. Blaž counted that they had 42 legs altogether. How many chickens were there?

When faced with a problem with contradicting data, pupils should realize that the information is contradictory and the problem with this information cannot be solved. For pupils in their early years of education, it is not easy to find where the obstacle that prevents a solution is, because it requires a great mental effort. If pupils rarely solve this kind of problems, they should be given a hint that the task includes contradictory information they need to look for. An example of such a problem is:

There are 24 pupils in the class. On a mathematics test, one third of them received a good grade, one third very good, one quarter excellent, 3 pupils received a sufficient grade, 1 pupil was absent during the test. How many students got an insufficient grade?

Methodology

Problem and objectives of the study

We wished to determine in the study whether the pupils who participate in our model of solving realistic problems in mathematical classes at primary level of basic school, which include problem situations from their life experiences, are more successful in solving realistic problems and thus also in constructing mathematical literacy. To this purpose we composed two written tests: with one the successfulness of solving realistic problems was tested at the beginning, and with the other at the end of the school year.

Research hypothesis

The experimental group will be more successful solving realistic problems with redundant data, with insufficient data, with multiple solutions, and with contradicting data than the control group.

Method

The causal – experimental method was applied in our research study. We investigated the influence of novelty, i.e. of integrating problem situations, or realistic problems with appropriate procedures, into the teaching of mathematics.

The sample

In the experiment 134 fourth grade basic school pupils were involved. The pupils were divided into 2 groups: in the experimental group (EG) there were 66 pupils, and in the control group (CG) 66 pupils. We produced the baseline and the final knowledge test for the purpose of this research by ourselves. The syllabus and the objectives defined in it were taken into account. The baseline testing was carried out at the beginning of the school year, and the final test at the end of the school year.

The experimental group enjoyed the whole range experimental treatment that included realistic mathematical problems, the strategies of solving a realistic mathematical problem, and modelling.

The variables were the attainments of the children in solving realistic problems:

- with redundant data,
- with insufficient data,
- with multiple solutions, and
- with contradicting data.

Results and interpretation

We interpreted the results in consistence with proving the set hypotheses. In testing the hypotheses the rule was observed the greatest risk for the rejection of a hypothesis allowed is 5%, the selected value for the level of significance is thus 0.05.

At the beginning of grade four (the baseline situation) there were no statistically significant differences in the attainments of the children in the experimental and in the control group.

Let us also present the basic statistical parameters of the final test (at the end of the experiment).

Table 1. The basic statistical parameters of the end test

	Group	***n***	**Attainment (%)**	**Arithmetic mean**	**Standard deviation**	**Min**	**Max**
Redundant data	EG	66	73.20	3.66	1.254	0.00	5.00
	CG	68	47.20	2.36	1.407	1.00	5.00
Insufficient data	EG	66	91.50	3.66	0.669	0.67	4.00
	CG	68	81.25	3.25	0.952	0.00	4.00
Multiple solutions	EG	66	68.63	5.49	1.895	0.25	8.00
	CG	68	45.13	3.61	2.280	0.00	7.75
Contradicting data	EG	66	71.80	3.59	1.105	0.25	5.00
	CG	68	64.00	3.20	1.128	0.50	5.00

Comparing the differences in the arithmetic means of all variables between EG and CG (Table 1), we can conclude EG was more successful in solving all types of realistic problems than CG.

With Leven's test of the equality of variances and with the *t*-test we checked in which variables the differences between the two groups were statistically significant at the end of the experiment.

Table 2. Presentation of the differences in the successfulness of solving realistic problems in EG and CG at the final test

Leven's test of the equality of variances			***t*-test**	
	F	**p**	**t**	**p**
Redundant data	0.004	0.949	5.516	0.000
Insufficient data	5.752	0.018	*2.995	0.004
Multiple solutions	0.285	0.594	5.040	0.000
Contradicting data	0.458	0.500	2.031	0.044

Note: *the Cochran-Cox approximation method of t-test was applied.

It is evident from Table 2 there is a statistically significant difference between the two groups in all the four variables.

The pupils in EG and in CG were the least successful in solving realistic *problems with multiple solutions* (in the EG the attainment was 68.63%, and in the CG 45.13%), as in the majority of cases they were satisfied with one solution, although the task stated they are expected to put down all possible solutions. Most pupils are namely convinced they do not need to search for other solutions if one is found, as they believe if one solution is found, the task is successfully solved.

It showed with the *problems that contain more data than necessary for the solution* as many as 73.20% of pupils in the EG read the text with understanding and were able to find the data needed for the solution. In the CG this problem was successfully solved by only 47.20% of pupils. They arrived at incorrect solution mainly because in solving they used all the data, including the redundant ones, as in the problems they had encountered in the classroom, they had always "had to" use all the data.

The pupils were also very successful in solving *problems that did not contain the number of data sufficient for solution,* as this task was successfully solved by as many as 91.50% of pupils from the EG and 81.25% of those from CG. Finally they also encountered problems with contradicting data. These problems were successfully solved by 71.80% of pupils in the EG and 64% of pupils in the CG. It showed for the pupils who have difficulties in mathematics it is very hard to find where the obstacle is that prevents the solution. We nevertheless proved with our study with an appropriate model of teaching and learning a critical attitude towards the data can be developed, as at the end of the experiment there was a statistically significant difference between the CG and the EG to the favour of EG.

Conclusion

We confirmed with our study how very important and necessary it is at he beginning of schooling to lead or to guide pupils when they encounter the solving of realistic problems. In this way they constantly develop and upgrade appropriate strategies of solving realistic problems in which their knowledge and skills are engaged. The pupils must participate in the conversation, from which their knowledge and understanding of mathematics, as well as their ways of thinking and reasoning and justifying the selected methods and procedures of solving problems can be identified. Interactive teaching and learning in which all the pupils in the class are involved proves to be as efficient.

It would probably be necessary not just to make corrections to the syllabus for mathematics, but to the system of learning and teaching mathematics, as today it is no longer sufficiently efficacious, and it does not meet the most urgent needs of humans in modern society. Some teachers erroneously offer the interpretation in mathematics classes emphasis should only be put on the phases of solving problems in which mathematical knowledge is involved. Realistic problems are so closely connected with life experience with their informal knowledge, experience, and previous mathematical knowledge pupils must pass through all the phases of solving a mathematical problem thus concurrently improving their mathematical literacy and raising its level. Pupils must be offered enough opportunities for more thorough integration of mathematics with everyday life and for continuously giving meaning to mathematical terms, concepts, and procedures in the phases of solving a realistic problem. It is the "absence" of giving meaning to terms, concepts, and procedures that leads to short-term memory and the reproduction of knowledge that in fact is not knowledge at all, but a kind of data or information that disappears after a relatively short period of time.